SMALL HYDRO CONDUIT OPTIMIZATION WITH DIFFERENTIAL CALCULUS.

by

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ABSTRACT.

The usual method of optimizing a powerplant conduit is to estimate the capital cost and add this to the present worth of the lost benefits, for a range of conduit sizes. This proceedure is time consuming since it is often necessary to estimate costs and benefits for 6 or more conduit sizes in order to reach the optimum. An alternative proceedure is to develop an equation for costs and lost benefits as a function of conduit diameter, as proposed by Fahlbusch (1), differentiate this equation with respect to diameter, equate to zero, and solve for diameter to obtain the optimum. This paper provides all nesessary equations in the 9-step proceedure, includes a method to take into account diferent operating modes - peaking or base loaded, and also includes two worked examples, one based on a proposed powerplant in the Northwest Territories, and the second on a high pressure steel penstock for an impulse unit in South America.

1. STEP # 1 - ASSEMBLE ALL DATA.

The following data is required. To illustrate the analysis, values for the various factors are provided beside each factor, to be used in the worked example. Where values are not provided for d, F, and R, these are calculated in later steps.

- c = 0.74 = Plant capacity factor, decimal.
- d = Water conduit diameter, in meters.
- e = 0.875 = Average plant efficiency, decimal.
- F = Conduit friction head loss, in meters.
- h = 22.5 = Turbine rated head, in meters.
- I = 7% = Real interest rate for present worth.
- L = 370 = Conduit length, in meters.
- n = 0.012 = Conduit friction coefficient, decimal.
- P = \$ 900 = Value of capacity, \$/kW.
- q = 49 = Average annual flow in $m^3.sec^{-1}$.
- Q = 66 = Rated turbine flow in $m^3.sec^{-1}$.
- R = Plant operating loss coefficient, decimal.
- W = \$ 0.10 = Value of energy in \$/kWh.
- Y = 60 = Plant life in years.

2. STEP # 2 - CALCULATE PLANT OPERATING LOSS COEFICIENT.

The plant capacity factor can vary from a high of about 0.75 for a base loaded plant to a low of about 0.2 for a peaking plant, and the daily load pattern can vary widely from operation at full load for only a few hours, with the remainder of the time at part load in a base loaded plant, to operation at full load for only a few hours in a peaking plant. Since the hydraulic losses are proportional to the flow squared, the losses are a function of the daily load pattern. The plant operating loss coefficient is defined as the ratio of the average daily loss divided by the loss at full load, if the plant were operated for a time equal to 8760 hours times the plant capacity factor, calculated from:-

$$R = \Sigma \text{ Sum of flow squared by time of flow duration, divided by capacity factor}$$

$$R = \Sigma Q^2_t \times t / c \qquad 1.$$

Where t = the number of hours the plant can be operated at full load, based on the available water supply. For example, a plant with a capacity factor of 0.6 could be operated for 60% of the time at 100% flow, would have a loss coefficient of:-

$$R = 1^2 \times 0.6 / 0.6 = 1$$

On the other hand. the plant could theoretically be operated at 60% flow for 100% of the time, with a loss coefficient of:-

$$R = 0.6^2 \times 1 / 0.6 = 0.6$$

In this case the loss coefficient is equal to the plant capacity factor. In practise, the loss coefficient will be somewhere between these two extremes, closer to the lower limit in high capacity factor plants and closer to the upper limit in peaking plants. For the worked example, the proposed powerplant will have 2 units supplying power to an isolated community. Both units are not expected to be operated jointly at full load, due to the lack of load, and are expected to operate as follows:-

	0.4 t	at	0.85 Q	= 0.34 Qt.	=	$0.289\ Q^2t$
	0.2 t	at	0.75 Q	= 0.15 Qt	=	$0.1125\ Q^2t$
	0.4 t	at	0.63 Q	= 0.25 Qt	=	$0.1588\ Q^2t$
Totals	1.0 t			0.74 Qt		$0.5603\ Q^2t$

In this case the plant can operate for 74% of time at full load and thus $R = 0.56 / 0.74 = 0.755$

3. STEP # 3 - DETERMINE PRESENT WORTH OF ANNUAL LOST INCOME PER METER OF HEAD LOSS.

The average annual income from the production of energy, per meter of head is:-

$$\text{Income (\$)} = 9.81 \times q \times 1.0 \times e \times 8760 \times W \qquad 2.$$

The total value of lost energy would then be the value of (2) times the present worth factor. With a real interest rate of 7% and a plant life of 60 years, the present worth factor is 14.04,. To continue with the worked example, energy loss value is:-

$$\text{Value} = 9.81 \times 49 \times 1 \times 0.875 \times 8760 \times 0.10 \times 14.04 = \$\ 5.173 \text{ million.}$$

If there is a value placed on lost capacity, then this can be calculated for one meter of head from:-

$$\text{Capacity value (\$)} = 9.81 \times Q \times 1.0 \times e \times P \qquad 3.$$

To continue with the example, assuming a diesel life of 20 years, replacement would take place in years 20 and 40. The present worth factor of this at 7% interest is $= 1 + 1.07^{-20} + 1.07^{-40} = 1.325$. Capacity value is then:-

$$\text{Value} = 9.81 \times 66 \times 1.0 \times 0.875 \times \$900 \times 1.325 = \$\ 0.675 \text{ million.}$$

The total value of one meter of head loss is then = \$ 5.173 + 0.675 = \$ 5.848 million.

4. STEP # 4 - DETERMINE A PRELIMINARY DIAMETER BY FORMULA.

To cost the conduit, a preliminary diameter is required, and for accuracy, this preliminary diameter should not be too far off the most economic diameter. There is a rule of thumb which states that the head losses in a high capacity factor plant should be in the region of 3% to 4% and in a low capacity factor plant, in the region of 6% to 8%. A simple formula which expresses this rule is:-

$$F = 0.08 \times (1 - c) \times h \qquad 4.$$

This formula results in a plant having a capacity factor of 0.2, would have hydraulic losses equal to 6.4% of head, and at a capacity factor of 0.6, the hydraulic losses would reduce to 3.2% of head. Formula 4 can then be combined with the Manning formula for hydraulic losses, and assuming a friction coefficient of 0.0115 as being representative of steel or concrete, the following formula can be derived for the diameter of a conduit:-

$$d = 0.466 \times Q^{0.37} \times L^{0.19} \times h^{-0.19} \times (1 - c)^{-0.19} \qquad 5.$$

However, this formula does not work when the ratio of conduit length to head is less than about 6. In such cases, the head loss through the trashracks and intake become relatively high, compared to the friction loss. Hence, for short conduits, the formulae developed by Fahlbusch (1) or Sarkaria (2) should be used, namely:-

$$d = 0.52 \times kW^{0.43} \times h^{-0.57} \qquad 6.$$

where kW = plant capacity. By combining equation 6 with the power formula kW = 9.81 Q h e, and assuming an overall turbine - generator efficiency of 90%, formula 6 can be re-written as:-

$$d = 1.33 \times Q^{0.43} \times h^{-0.14} \qquad 7.$$

Depending on the conduit to head ratio, either formulae 5 or 7 can be used to obtain a preliminary conduit diameter. To continue with the example, with a length to head ratio = 370 / 22.5 = 16.4 , use formula 5 to determine the diameter to be used for costing as follows:-

$$d = 0.466 \times 66^{0.37} \times 370^{0.19} \times 22.5^{-0.19} \times (1 - 0.742)^{-0.19} = 4.83 \text{ m.}$$

5. STEP # 5 - DETERMINE HEAD LOSS AS A FUNCTION OF CONDUIT DIAMETER.

The Manning equation for friction losses in a circular conduit can be re-written as:-

$$F(max) = 10.4 \times Q^2 \times n^2 \times L \times d^{-5.33} \qquad 8.$$

Since Q, n and L are all known quantities, the head loss becomes a function of the diameter. Other formulae for friction loss can be substituted, but the Manning equation is the simplest to use. Equation 6 will give the head loss at full flow. Since the plant could be operated at part load, the full load head loss has to be adjusted to reflect the operating mode. This is accomplished by multiplying by the plant operating loss coefficient R. Hence:-

$$F(average) = 10.4 \times R \times Q^2 \times n^2 \times L \times d^{-5.33} \qquad 9.$$

This is an important step, since it could result in a smaller economic diameter, thus improving the economic viability of the project. For instance, in the worked example, the cost saving is almost 8% when the operating mode is taken into account. The powerplant will have a concrete lined tunnel of inverted D shape, and the head loss, as a function of diameter is:-

$$F(max) = \{ Q \times n \times (0.893\, d^2)^{-1} \times (0.25\, d)^{-1} \}^2 \times L$$

With Q = 66, n = 0.012 and L = 370, the friction loss formula becomes $F = 1850\ d^{-5.33}$, adjusted for the flow pattern

$$F(average) = 0.755 \times 1850\, d^{-5.33} = 1397\, d^{-5.33}$$

6. STEP # 6 - DETERMINE CONDUIT COST FOR THE PRELIMINARY DIAMETER.

From the project drawings, determine the cost of the water conduit, using the preliminary diameter determined in step 4. Include in the cost any appurtenances associated with the conduit such as a surge tank. The cost of the conduit can be assumed to be a function of the diameter squared. For a fixed length, excavation volumes vary with diameter squared, and concrete or steel linings, for the same pressure head and length, also vary with the diameter squared.

The conduit cost should then be converted into an equation which is a function of the diameter squared, by dividing the estimated cost by the diameter squared. If the conduit has several diameters, select an average representative diameter. If the analysis indicates that the most economic diameter is, say, 10% larger than the costed representative diameter, then increase all diameters by 10%.

For the example, it was decided to estimate the cost of a 4.9 meter inverted D shaped concrete lined tunnel, surge tank and very short penstock. This was found to total \$ 5.3 million, not including any overhead costs such as contractor's move-in or move-out expenses, camp cost, engineering, contingencies and interest. In other words, only the direct construction cost was estimated. This was to allow for the fact that for a small change in the volume of work, the change in cost is lower than the multiple of the change in volume times the unit price. Expressing the cost as a function of the diameter squared:-

$$\text{Cost} = \$\,5.3 \text{ divided by } 4.9^2 = \$\,0.221\ d^2 \text{ million.}$$

7. STEP # 7 - DETERMINE PRESENT WORTH OF HEAD LOSS AS A FUNCTION OF THE DIAMETER.

This is simply the product of the total present worth (equations 2 + 3) per meter of head loss, times the head loss as a function of diameter from equation 9. The example value is then:-

$$\text{Value} = \$\,5.848 \times 1397 \times d^{-5.33} \text{ million.} = \$\,8{,}169 \times d^{-5.33} \text{ million.}$$

8. STEP # 8 - DETERMINE TOTAL COST (PRESENT WORTH + CONSTRUCTION) AS FUNCTION OF DIAM.

The total cost is the addition of the total present worth determined in step 7, plus the construction cost determined in step 6. For the example, this becomes:-

$$\text{Total cost} = \$\,8{,}169\, d^{-5.33} \times 10^6 + 0.221\, d^2 \times 10^6.$$

9. STEP # 9 - DIFFERENTIATE AND EQUATE TO ZERO.

The equation obtained in step 8 is now differentiated with respect to diameter and then equated to zero to obtain the minima. The resulting equation is solved for d to obtain the most economic diameter. Other more complex conduits can be optimised in a simmilar manner. Where there is - for example - a tunnel followed by a long penstock, each should be optimized independently. In other cases where the water conduit is relatively short, and the intake comprises a large proportion of the cost, the intake should be included in the cost, with the cost being a function of the diameter cubed. This adds another term to the differential equation, but does not prevent solving for the diameter.

Returning to the example, differentiate the total cost equation and equate to zero:-

$$\Delta \text{Cost} / \Delta d = \{ -5.33 \times 8{,}169\, d^{-6.33} + 2 \times 0.221\, d \} \times 10^6 = 0$$
$$d^{7.33} = 5.33 \times 8{,}169 \times 2^{-1} \times 0.221^{-1} = 98{,}508$$
$$d = 4.80 \text{ meters.}$$

The economic diameter is then 4.8 meters, or slightly less than the preliminary diameter of 4.83 meters calculated in step 4. The close agreement between preliminary and final diameters is a coincidence in this example. Usually the difference is larger. Finally, to see what effect inclusion of the overhead costs would have on the economic diameter, these were added to the cost. For the remote location of the powerplant, in the Northwest Territories, overhead costs are very high as a proportion of the direct cost. The cost was found to increase by 60% with the addition of contractor's move in-out expenses, camp operation, engineering and interest. This would result in a tunnel diameter of 4.50 meters or just over 6% smaller, for a small cost saving of less than 2% of the total project cost. However, the author is of the opinion that such an interpretation of the analysis will result in too small a tunnel diameter, since overhead costs are not directly tied to the volume of work.

10. PENSTOCK STEEL CASE WITH DIRECT DIFFERENTIATION.

For the second worked example, the author has selected a high head powerplant in the Andes of South America, to illustrate the direct use of differentiation in a case where the construction cost is a function of the diameter. The plant will have a 700m. long horizontal steel penstock in a tunnel leading to an undeground powerhouse containing two impulse turbines operating under a rated head of 1022m., to which 25% should be added for waterhammer at the downstream end of the penstock. Average waterhammer on the 700m. long pipe is 20%. Rated flow is 7 cumecs. Plant capacity factor is 0.2 and the operating loss coefficient is 1.0. Turbine-generator efficiency is 0.882. Capacity is valued at $75 per annum per kW and energy at $0.0512 per kWh. Present worth factor is 9.97 based on a plant life of 60 years and a real interest rate of 10%. Cost of the steel penstock is $5.00 per kg., installed.

At this point the author assumes that the reader is familiar with the various steps. In the interest of saving space, the steps are eliminated, and the source of the numbers in the calculations can be traced by their values.

A preliminary diameter for the penstock can be calculated from formula 7 as follows:-

$$d = 1.33 \times 7^{0.43} \times 1022^{-0.14} = 1.16 \text{ m.}$$

Manning's formula should not be used for small pipes since it underestimates the friction loss. Instead use the metric version of the Hazen-Williams formula, which can be derived from data by Merrit (3) as follows:-

$$F = 10.8 \times C^{-1.85} \times L \times Q^{1.85} \times d^{-4.87} \qquad 10.$$

where C = friction coefficient, which has a value of 115 for welded steel pipe. Head loss due to friction is then:-

$$F = 10.8 \times 115^{-1.85} \times 700 \times 7^{1.85} \times d^{-4.87} = 42.62 \times d^{-4.87}$$

And the present worth of energy and capacity will be:-

$$\text{P.W. energy} = \$\,9.97 \times 9.81 \times 7 \times 42.62 \times d^{-4.87} \times 0.882 \times 8760 \times 0.2 \times 0.0512 = \$\,2{,}308{,}500 \times d^{-4.87}$$

$$\text{P.W. capacity} = \$\,9.97 \times 9.81 \times 7 \times 42.62 \times d^{-4.87} \times 0.882 \times 75 = \$\,1{,}930{,}200 \times d^{-4.87}$$

$$\text{Total P.W.} = \$\,4{,}238{,}700 \times d^{-4.87}$$

To determine the cost of the penstock, the weight of steel needs to be calculated. The penstock thickness formula is:-

$$t = 9.81 \times d \times h \times (2 \times S \times J)^{-1} + \text{corrosion allowance.} \qquad 11.$$

Where t = steel thickness in mm. J = weld joint efficiency = 1.0
S = allowable steel stress in MPa. h = pressure head including waterhammer

For such a high head penstock, high strength quenched and tempered steel would be used to reduce weight. The allowable stress for Q&T steel is 263 MPa. (5). Corrosion allowance should be 1.5 mm., which can be simulated with a thickness increase of 5%, based on the preliminary diameter and thickness from formula 11. Average pipe thickness is then:-

$$t = 9.81 \times d \times 1022 \times 1.20 \times 0.5 \times 263^{-1} \times 1.05 = 24.02 \times d \text{ mm.}$$

and cost, based on steel weight of 7.85 kg per square meter per millimeter thickness, $ 5.00 per kg. will be:-

$$\text{Cost} = \$\,\pi \times d \times 700 \times 24.02 \times d \times 7.85 \times 5 = \$\,2{,}073{,}300 \times d^2$$

Total of construction cost and present worth of benefits is then:-

$$\text{Cost} = \$\,2{,}073{,}300 \times d^2 + 4{,}238{,}700 \times d^{-4.87},$$

and differentiating with respect to diameter:-

$$\Delta\,\text{Cost} / \Delta\,d = 2 \times 2{,}073{,}300 \times d - \{ 4.87 \times 4{,}238{,}700 \times d^{-5.87} \}$$

$$d^{6.87} = 4.87 \times 4.2387 \times (2 \times 2.0733)^{-1} = 4.9782 \text{ ------------------------- and } d = 1.26 \text{ m.}$$

Maximum penstock steel thickness is then $t = 9.81 \times 1.26 \times 1022 \times 1.25 \times 0.5 \times 263^{-1} + 1.5 = 31.5$ mm.

In this case the most economic diameter is 8.6% larger than that determined from the Fahlbush formula. For interest, this result can be compared with that obtained from the equation 1-15 given in reference 4, page 19, at 1.20m. The smaller economic diameter is probably due to use of the Darcy-Weisbach friction equation which produces a lower estimate of friction loss. (Note that equation 1-15 contains a typographical error, the constant should be 0.5, not 0.05.)

11. CONCLUSIONS.

The two examples illustrate how differential claculus can simplify the calculation of the economic conduit diameter, by eliminating the multiple cost estimates required in the graphical approach. However, it must be remembered that the diameter selected for the preliminary cost estimate, must be reasonably close to the economic diameter, and this is where formulae #6 and #7 will prove most useful.

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